International conference
ANALYTICAL METHODS OF CELESTIAL MECHANICS

Adiabatic approximation in dynamical studies of exoplanetary systems with mean-motion resonances

V.V.Sidorenko

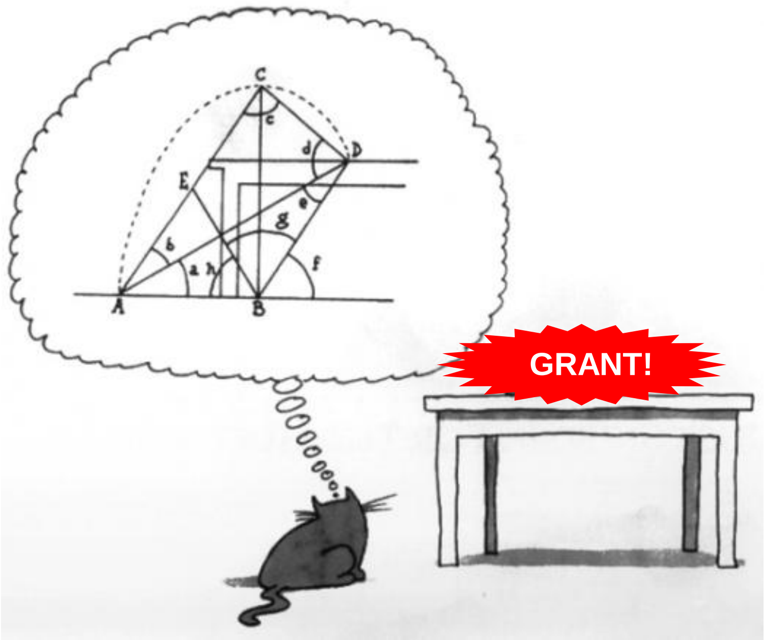
Keldysh Institute of Applied Mathematics RAS

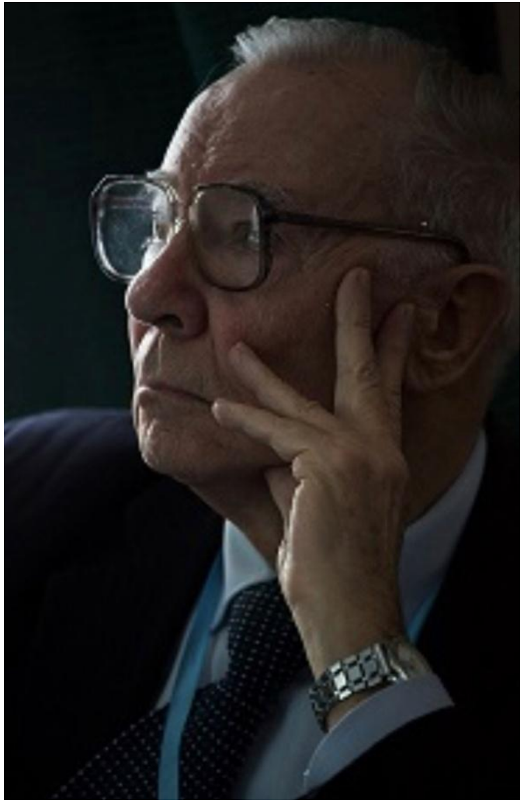
vvsidorenko@list.ru

Euler Institute, August 19-23, 2024.

Outline

1. A few words in memory of K.V.Kholshevnikov
2. Evolutionary celestial mechanics as a section of analytical celestial mechanics. Key stages of its development. Peculiarites of studying secular effects in exoplanetary systems
3. Adiabatic approximation in studies of mean-motion resonances
4. Example: co-orbital motion
5. Conclusion





K.V.Kholshevnikov
1939-2021

Evolutionary Celestial Mechanics

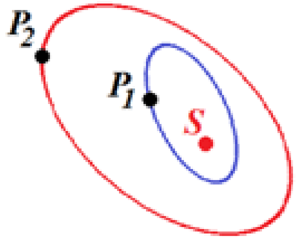
ECM: the study of the evolution of the motion of celestial bodies over long time intervals (significantly exceeding the typical value of the orbital period of the system under study)

Analytical approaches
(the main object of the study is the three-body problem)

Numerical approaches
(complexity of models is limited by performance of computers)

Formation of paradigms (e.g. recognition of the importance of the Lidov-Kozai effect)

Main model: three-body problem

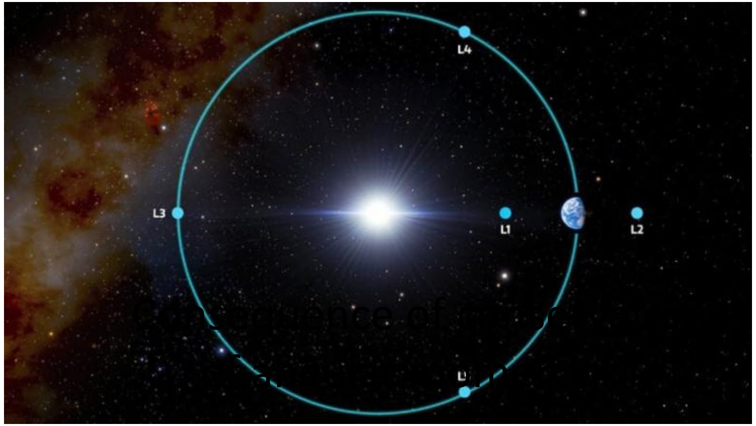


$$m_0 >> m_i \quad (i = 1, 2)$$

The situations studied by celestial mechanics are characterized by the presence of a dominant body!

Studied since the XVIIth century!

First results concerning resonant motions: Euler & Lagrange.



Evolutionary celestial mechanics in recent decades

1980s: chaos (Wisdom & Laskar), non-gravitational effects (e.g.. Yarkovsky effect)

2000s: rapid increase in the number of known objects, transition from studies of the dynamics of individual objects to the dynamics of populations and analysis of dynamic structures, the emergence of new objects of study (exoplanetary systems)

Exoplanetary systems

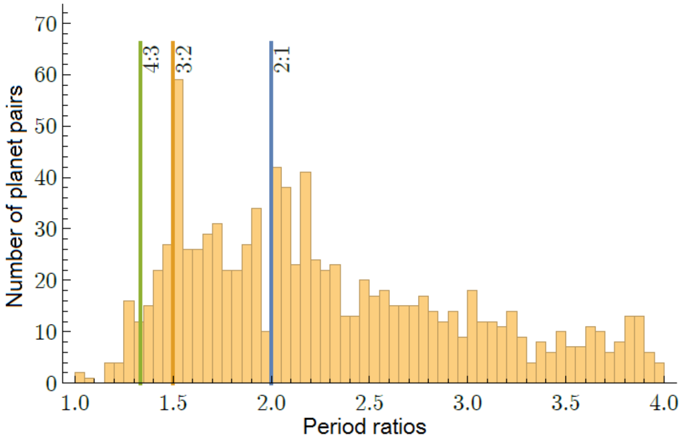
The first reliably discovered exoplanet : 1992
At this moment (August 08, 2024): 5743 confirmed exoplanets in 4286 planetary systems;
961 multiple planet systems;
Nobel Prize in Physics for the discovery of the first exoplanet around a Sun-like star: 2019 (M.Mayer, D.Quelez)

Some remarks about studies of secular effects in dynamics of exoplanetary systems

The usual model: the general three-body problem.

Analysis of motions that were previously considered physically impossible (for example, planets in counter-rotation).

Frequently realized resonant modes of motion.



Mean motion resonance (MMR) is the dynamical situation where the ratio of the orbital periods of two orbiting objects is close to the ratio of two small integers

Number of known first-order MMR in exoplanetary systems (taking from Pichierri 2019)

How is the adiabatic approximation applied in MMP studies?

Time scales at the resonance (planar problem)

T_1 - orbital motions periods

T_2 - timescale of rotations/oscillations of the resonant argument

T_3 - secular evolution of eccentricities e and longitudes of perihelion ω

$$\boxed{T_1 \ll T_2 \ll T_3}$$

Strategy: 1. Averaging of the orbital motions, taking into account MMR;

2. Analysis of the auxiliary 1DOF Hamiltonian system describing the variation of the resonant argument ;

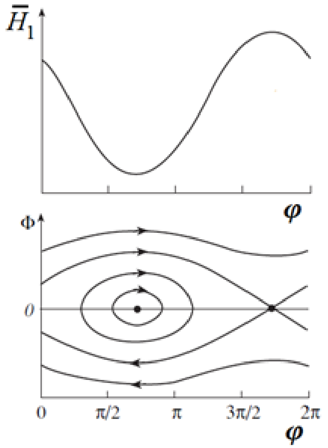
3. Averaging the right-hand sides of the equations for slow variables along the variations of the resonant argument

Adiabatic approximation

4. Integrable limit (e=0): X - 1DOF Hamiltonian system,

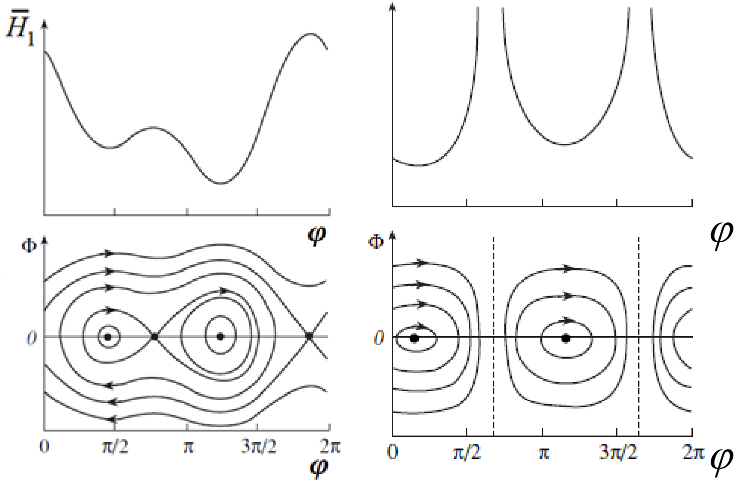
x, y - parameters

If we are lucky:

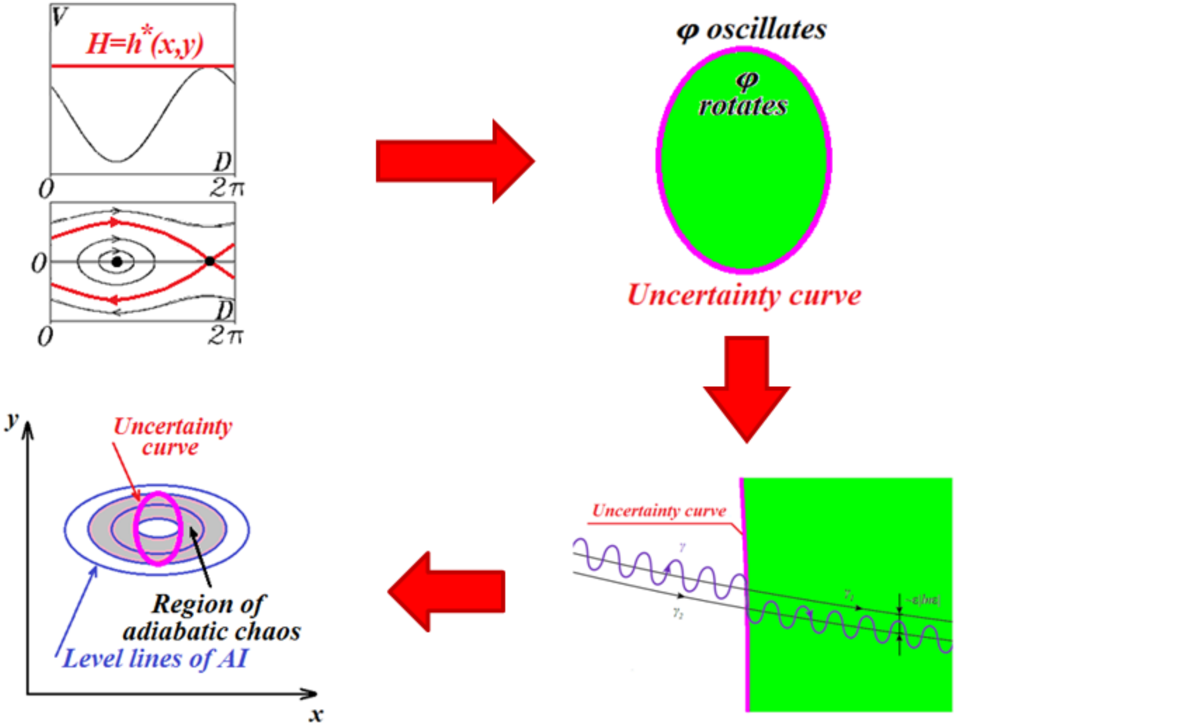


Phase portrait topologically equivalent to phase portrait of mathematical pendulum

If we are unlucky:



Adiabatic approximation



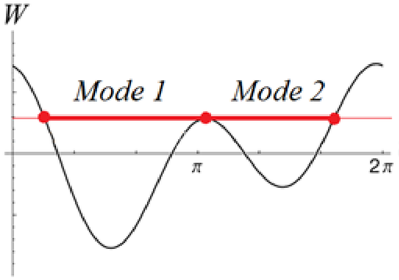
The formation of the region of adiabatic chaos

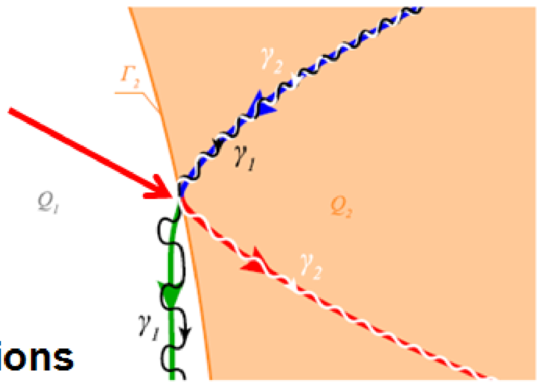
The jump between solutions of the averaged equations in the vicinity of UC

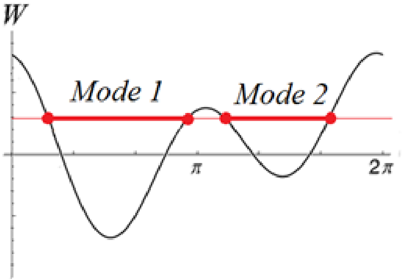
Neishtadt, A.I.: Jumps of the adiabatic invariant on crossing the separatrix and the origin of the 3:1 Kirkwood gap. Sov. Phys. Dokl. **32, 571–573 (1987)**

2. Adiabatic approximation: basic ideas

Geometric chaos

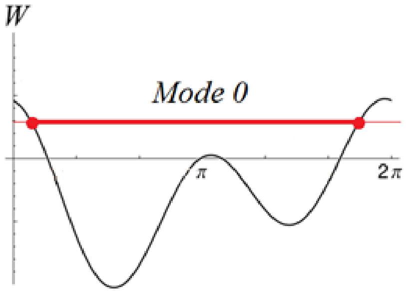






Probabilities of transitions between modes:

Artemyev et al. (2013)



Neishtadt, A., Treschev, D.: Polymorphisms and adiabatic chaos. Ergodic Theory & Dynamical Systems. 31, 259-284 (2011)

↓

Markov chains!

Adiabatic approximation: Wisdom (1985)

1. A.I.Neishtadt, V.V.Sidorenko: Wisdom system. Dynamics in adiabatic approximation. CMDA, 90, 307-330 (2004)
2. V.V.Sidorenko: Evolution of asteroid orbits at the 3:1 their mean motion resonance with Jupiter (planar problem). Cosmic Research, 44, 440-455 (2006)
3. V.V. Sidorenko, A.I. Neishtadt, A.V. Artemyev, L.M. Zelenyi: Quasi-satellite orbits in the general context of dynamics in the 1: 1 mean motion resonance: perturbative treatment. CMDA, 120,131-162 (2014)
4. S.S.Efimov, V.V.Sidorenko: On asymmetric Zeipel-Lidov-Kozai cycles in mean motion resonaces. Cosmic Research, 58, 249-255(2020)
5. S.S.Efimov, V.V.Sidorenko: An analytically treatable model of long-term dynamics in mean motion resonance with coexisting resonant modes. CMDA, 132(5), 1-28 (2020)
6. V.V.Sidorenko: Dynamics of “jumping” Trojans. Perturbative treatment. CMDA, 130(10), 1-18 (2018)
7. V.V.Sidorenko: A perturbative treatment of retrograde co-orbital motion. Astron. J., 160 (6) (2020)

Co-orbital motion

D: co-orbital motion of two celestial bodies means their 1:1 MMR in orbiting a central body (Funk, Dvorak & Schwarz,2017)

Co-orbital motion: exchange orbits (Laughlin & Chambers, 2002)

A-exchange orbits

Ex: Janus & Epimetheus

E-exchange orbits

Consequence of conservation of angular momentum

Co-orbital exoplanets

Theoretical possibility:

Laughlin & Chambers 2002,
Beauge et al 2007, Cresswell & Nelson 2009

Lack of identification

Formation constraints?

Constraints to detectability?

Bulletin of the AAS • Vol. 54, Issue 5

TOI-178: a window into the formation and evolution of planetary systems

ert[2] Nathan Hara[3]
mas G Wilson[5]
C C Barros[6] Nicolas Billot[3]
ndeker[9] Giovanni Bruno[10]
ry[2] David Ehrenreich[3]
ridlund[14]
itzmann[2] Baptiste Lavie[3]
is[2] Hugh Osborn[2]
rancisco J Pozuelos[17]
Schneider[8]

Geneva,
ws, [6]IA-Univ. of Porto,
iversity,
oimbra,
rs University of Technology, [15]LAM,
ica de Andalucia,

A&A 624, A46 (2019)
https://doi.org/10.1051/0004-6361/201834901

Astronomy & Astrophysics

Co-orbital exoplanets from close-period candidates: the TOI-178 case

A. Leleu[1,⋆], J. Lillo-Box[2], M. Sestovic[3], P. Robutel[4], A. C. M. Correia[5,4], N. Hara[6,⋆], D. Angerhausen[3,7], S. L. Grimm[3], and J. Schneider[8]

[1] Physikalisches Institut, Universität Bern, Gesellschaftsstr. 6, 3012 Bern, Switzerland
e-mail: adrien.leleu@space.unibe.ch
[2] European Southern Observatory, Alonso de Cordova 3107, Vitacura Casilla 19001, Santiago 19, Chile
[3] Center for Space and Habitability, University of Bern, Gesellschaftsstr. 6, 3012 Bern, Switzerland
[4] IMCCE, Observatoire de Paris – PSL Research University, UPMC University Paris 06, University Lille 1, CNRS, 77 Avenue Denfert-Rochereau, 75014 Paris, France
[5] CFisUC, Department of Physics, University of Coimbra, 3004-516 Coimbra, Portugal
[6] Observatoire de Genève, Université de Genève, 51 ch. des Maillettes, 1290 Versoix, Switzerland
[7] Blue Marble Space Institute of Science, 1001 4th Ave Suite 3201, Seattle, WA 98154, USA
[8] Paris Observatory, LUTh UMR 8102, 92190 Meudon, France

Received 17 December 2018 / Accepted 1 February 2019

ABSTRACT

Despite the existence of co-orbital bodies in the solar system, and the prediction of the formation of co-orbital planets by planetary system formation models, no co-orbital exoplanets (also called trojans) have been detected thus far. Here we study the signature of co-orbital exoplanets in transit surveys when two planet candidates in the system orbit the star with similar periods. Such a pair of candidates could be discarded as false positives because they are not Hill-stable. However, horseshoe or long-libration-period tadpole co-orbital configurations can explain such period similarity. This degeneracy can be solved by considering the transit timing variations (TTVs) of each planet. We subsequently focus on the three-planet-candidate system TOI-178: the two outer candidates of that system have similar orbital periods and were found to have an angular separation close to $\pi/3$ during the TESS observation of sector 2. Based on the announced orbits, the long-term stability of the system requires the two close-period planets to be co-orbital. Our independent detrending and transit search recover and slightly favour the three orbits close to a 3:2:2 resonant chain found by the TESS pipeline, although we cannot exclude an alias that would put the system close to a 4:3:2 configuration. We then analyse the co-orbital scenario in more detail, and show that despite the influence of an inner planet just outside the 2:3 MMR, this potential co-orbital system could be stable on a gigayear time-scale for a variety of planetary masses, either on a trojan or a horseshoe orbit. We predict that large TTVs should arise in such a configuration with a period of several hundred days. We then show how the mass of each planet can be retrieved from these TTVs.

Key words. celestial mechanics – planets and satellites: detection – planets and satellites: dynamical evolution and stability

Preliminary part: motion equations (1)

$$\boxed{\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\mathbf{r}}_i}\right) - \frac{\partial \mathcal{L}}{\partial \mathbf{r}_i} = 0 \quad (i = 1, 2)}$$

$$\mathcal{L}(\dot{\mathbf{r}}_i, \mathbf{r}_i) = T(\dot{\mathbf{r}}_i) + V(\mathbf{r}_i)$$

$$T(\dot{\mathbf{r}}_i) = \frac{1}{2}\left\{\mu\left[\bar{\mu}_1(1-\mu\bar{\mu}_1)\dot{r}_1^2 + \bar{\mu}_2(1-\mu\bar{\mu}_2)\dot{r}_2^2\right] - 2\mu^2\bar{\mu}_1\bar{\mu}_2(\dot{\mathbf{r}}_1, \dot{\mathbf{r}})\right\}$$

Mass of the star: $1 - \mu$

Masses of the planets: $\mu\bar{\mu}_1,\ \mu\bar{\mu}_2 \quad (\bar{\mu}_1 + \bar{\mu}_2 = 1)$

$$V(\mathbf{r}_i) = \mu\left[\frac{\bar{\mu}_1(1-\mu)}{r_1} + \frac{\bar{\mu}_2(1-\mu)}{r_2}\right] + \frac{\mu^2\bar{\mu}_1\bar{\mu}_2}{|\mathbf{r}_1 - \mathbf{r}_2|}$$

$$\boxed{\frac{d\mathbf{p}_i}{dt} = -\frac{\partial \mathcal{H}}{\partial \mathbf{r}_i},\quad \frac{d\mathbf{r}_i}{dt} = \frac{\partial \mathcal{H}}{\partial \mathbf{p}_i} \quad (i = 1, 2)}$$

$$\mathbf{p}_i = (\partial T / \partial \dot{\mathbf{r}}_i), \qquad \mathcal{H}(\mathbf{p}_i, \mathbf{r}_i) = T(\mathbf{p}_i) - V(\mathbf{r}_i)$$

$$T(\mathbf{p}_i) = \frac{1}{2}\left[\frac{1}{\mu}\left(\frac{p_1^2}{\bar{\mu}_1} + \frac{p_2^2}{\bar{\mu}_2}\right) + \frac{1}{1-\mu}(\mathbf{p}_1 + \mathbf{p}_2, \mathbf{p}_1 + \mathbf{p}_2)\right]$$

"Democratic" variables

(Morbidelli (2002), Laskar & Robutel (1995))

$$\bar{\mathbf{p}}_i = \mathbf{p}_i / \mu, \quad \bar{\mathcal{H}}(\bar{\mathbf{p}}_i, \mathbf{r}_i) = \bar{\mathcal{H}}_0(\bar{\mathbf{p}}_i, \mathbf{r}_i) - \mu V(\bar{\mathbf{p}}_i, \mathbf{r}_i) + O(\mu^2)$$

$$\boxed{(\mathbf{p}_i, \mathbf{r}_i) \mapsto (\bar{\mathbf{p}}_i, \mathbf{r}_i)}$$

$$\bar{\mathcal{H}}_0(\bar{\mathbf{p}}_i, \mathbf{r}_i) = \bar{\mathcal{H}}_{01}(\bar{\mathbf{p}}_1, \mathbf{r}_1) + \bar{\mathcal{H}}_{02}(\bar{\mathbf{p}}_2, \mathbf{r}_2), \quad \bar{\mathcal{H}}_{0i}(\bar{\mathbf{p}}_i, \mathbf{r}_i) = \frac{\bar{p}_i^2}{2\bar{\mu}_i} - \frac{\bar{\mu}_i}{r_i} \quad (i = 1, 2)$$

Rescaling to eliminate singularity

(Robutel et al (2016))

$$V(\bar{\mathbf{p}}_i, \mathbf{r}_i) = \frac{\bar{\mu}_1\bar{\mu}_2}{|\mathbf{r}_1 - \mathbf{r}_2|} - \frac{1}{2}(\bar{\mathbf{p}}_1 + \bar{\mathbf{p}}_2, \bar{\mathbf{p}}_1 + \bar{\mathbf{p}}_2)$$

Preliminary part: motion equations (2)

$$\boxed{(\bar{\mathbf{p}}_i, \mathbf{r}_i) \mapsto (L_i, G_i, l_i, \varpi_i)}$$

$$L_i = \bar{\mu}_i \sqrt{a_i}, \qquad l_i,$$
$$G_i = L_i \sqrt{1-e_i^2}, \qquad \varpi_i.$$

$$\mathscr{H} = -\frac{\bar{\mu}_1^3}{2L_1^2} - \frac{\bar{\mu}_1^3}{2L_1^2} - \mu V(L_1, L_2, G_1, G_2, l_1, l_2, \varpi_1 - \varpi_2) + O(\mu^2)$$

Resonance argument: $\varphi = \lambda_1 - \lambda_2 = l_1 - l_2 + \varpi_1 - \varpi_2$

$$\boxed{(L_i, G_i, l_i, \varpi_i) \mapsto (P_\varphi, P_l, P_\Sigma, P_\Delta, \varphi, l, \varpi_\Sigma, \varpi_\Delta)}$$

canonical transformation with the valence c=2

$$L_1 = \frac{1}{2}\left[P_l + (P_\varphi - P_\varphi^*)\right], \; L_2 = -\frac{P_\varphi}{2},$$

$$G_1 = \frac{1}{2}(P_\Sigma + P_\Delta) + \frac{1}{2}(P_\varphi - P_\varphi^*), \quad G_2 = \frac{1}{2}(P_\Sigma - P_\Delta) - \frac{1}{2}(P_\varphi - P_\varphi^*), \quad P_\varphi^* = -2\bar{\mu}_2\sqrt{a_*}$$

$$l = l_1, \; \varpi_\Sigma = \varpi_1 + \varpi_2, \; \varpi_\Delta = \varpi_1 - \varpi_2$$

Now *l* is the only fast variable.
We can average over it!

Resonant approximation (1)

Scale transformation

$$\boxed{\Phi = (P_\varphi^* - P_\varphi) / 2\bar{\mu}_1\bar{\mu}_2\varepsilon, \qquad \tau = \varepsilon t}$$

$$\varepsilon = \sqrt{\mu}$$

$$W = \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{1}{|\mathbf{r}_1 - \mathbf{r}_2|} - (\dot{\mathbf{x}}_1, \dot{\mathbf{x}}_2)\right)dl_1$$

Slow-fast system

Fast variables

$$\frac{d\varphi}{d\tau} = 3\Phi, \qquad \frac{d\Phi}{d\tau} = -\frac{\partial W}{\partial \varphi}$$

$$\frac{dP_\Delta}{d\tau} = 2\varepsilon\bar{\mu}_1\bar{\mu}_2\frac{\partial W}{\partial \varpi_\Delta}, \qquad \frac{d\varpi_\Delta}{d\tau} = -2\varepsilon\bar{\mu}_1\bar{\mu}_2\frac{\partial W}{\partial P_\Delta}, \qquad \frac{d\varpi_\Sigma}{d\tau} = -2\varepsilon\bar{\mu}_1\bar{\mu}_2\frac{\partial W}{\partial P_\Sigma},$$

SF-Hamiltonian

$$\Xi = \frac{\chi\Phi^2}{2} + W(P_\Sigma, P_\Lambda, \varphi, \varpi_\Delta)$$

$$\omega = d\Phi \wedge d\varphi + (2\varepsilon\bar{\mu}_1\bar{\mu}_2)^{-1}dy \wedge dx$$

Resonance (q+k):q

$$\chi = 3(q+k)^{4/3}[\bar{\mu}_1(q+k)^{2/3} + \bar{\mu}_2 q^{2/3}]$$

$$q = 1, k = 0 \Rightarrow \chi = 3$$

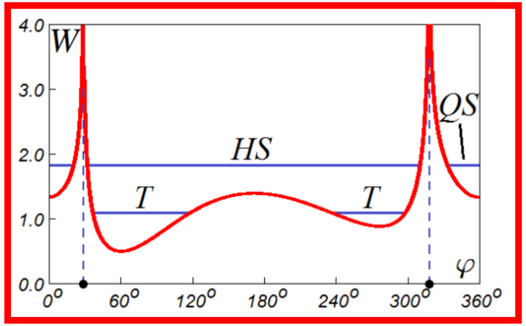
Resonant approximation (2)

$$a_* = 1,\ P_\Delta = \bar{\mu}_1\sqrt{1-e_1^2} - \bar{\mu}_2\sqrt{1-e_2^2},\ P_\Sigma = \bar{\mu}_1\sqrt{1-e_1^2} + \bar{\mu}_2\sqrt{1-e_2^2} = 1-\sigma$$

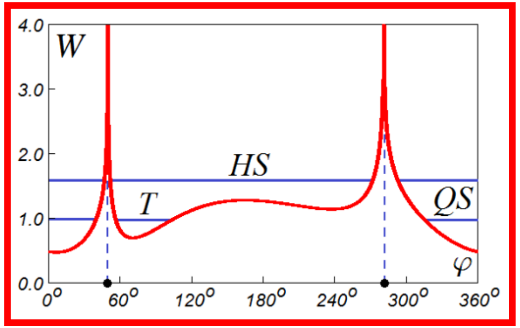
Angular momentum deficit

Integrable limit ($\varepsilon = 0$): Ξ - 1DOF Hamiltonian system,

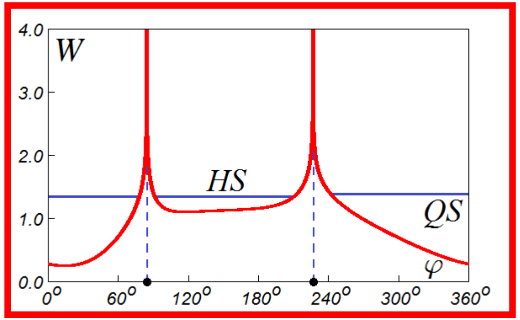
x, y - parameters



$e_1 = 0.3,\ \ e_2 = 0.3,\ \ \varpi_\Delta = 60^\circ$



$e_1 = 0.6,\ \ e_2 = 0.3,\ \ \varpi_\Delta = 60^\circ$



$e_1 = 0.9,\ \ e_2 = 0.3,\ \ \varpi_\Delta = 60^\circ$

$\varphi_1(e_1, e_2, \varpi_\Delta),\ \varphi_2(e_1, e_2, \varpi_\Delta)$ - resonant phase values at which W tends to infinity

Averaging over the fast subsystem solutions on the level $\Xi = \xi$

$$\dot{e}_1 = -\frac{\varepsilon\bar{\mu}_2\sqrt{1-e_1^2}}{e_1}\left\langle\frac{\partial W}{\partial \varpi_\Delta}\right\rangle, \quad \dot{e}_2 = \frac{\varepsilon\bar{\mu}_1\sqrt{1-e_2^2}}{e_2}\left\langle\frac{\partial W}{\partial \varpi_\Delta}\right\rangle,$$

$$\dot{\varpi}_1 = \frac{\varepsilon\bar{\mu}_2\sqrt{1-e_1^2}}{e_1}\left\langle\frac{\partial W}{\partial e_1}\right\rangle, \quad \dot{\varpi}_2 = \frac{\varepsilon\bar{\mu}_1\sqrt{1-e_2^2}}{e_2}\left\langle\frac{\partial W}{\partial e_2}\right\rangle.$$

$$\left\langle\frac{\partial W}{\partial \zeta}\right\rangle = \frac{1}{T(e_1, e_2, \varpi_\Delta, \xi)}\int_0^{T(e_1, e_2, \varpi_\Delta, \xi)} \frac{\partial W}{\partial \zeta}(e_1, e_2, \varpi_\Delta, \varphi(\tau, e_1, e_2, \varpi_\Delta, \xi))d\tau$$

$$\zeta = e_1, e_2, \varpi_\Delta$$

$$J(e_1, e_2, \varpi_\Delta, \xi) = \frac{3}{2\pi}\int_0^{T(e_1, e_2, \varpi_\Delta, \xi)} \Phi^2(\tau, e_1, e_2, \varpi_\Delta, \xi)d\tau$$

First integral of double averaged system

Now it is possible to draw phase portraits!

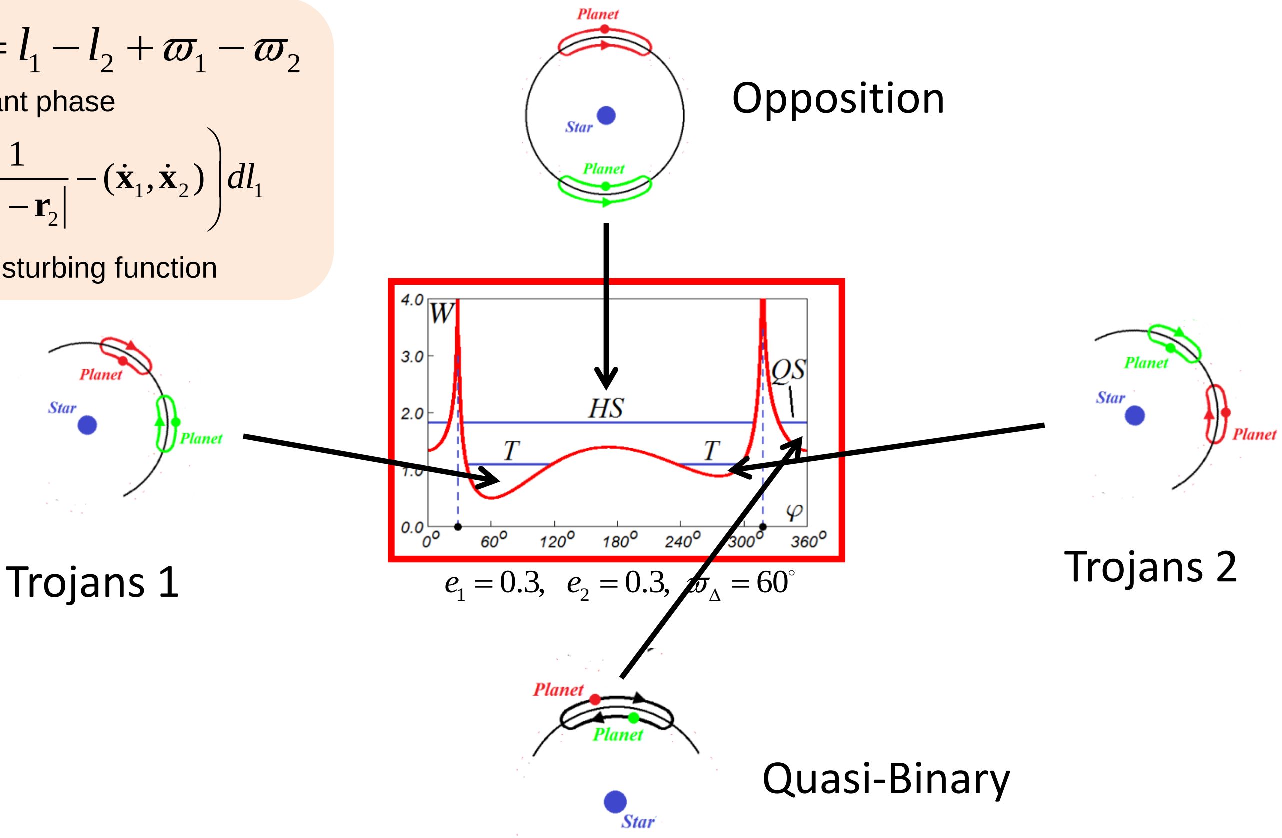
Co-orbital motions (planar priblem)

$$\varphi = \lambda_1 - \lambda_2 = l_1 - l_2 + \varpi_1 - \varpi_2$$

Resonant phase

$$W = \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{1}{|\mathbf{r}_1 - \mathbf{r}_2|} - (\dot{\mathbf{x}}_1, \dot{\mathbf{x}}_2)\right) dl_1$$

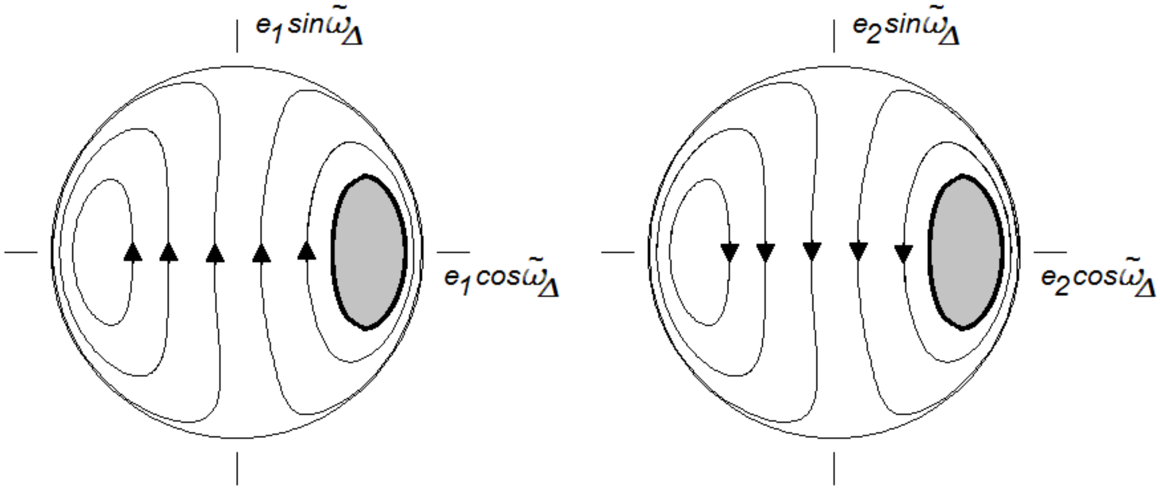
Averaged disturbing function



$e_1 = 0.3,\quad e_2 = 0.3,\quad \varpi_\Delta = 60^\circ$

Secular evolution of co-orbital motion

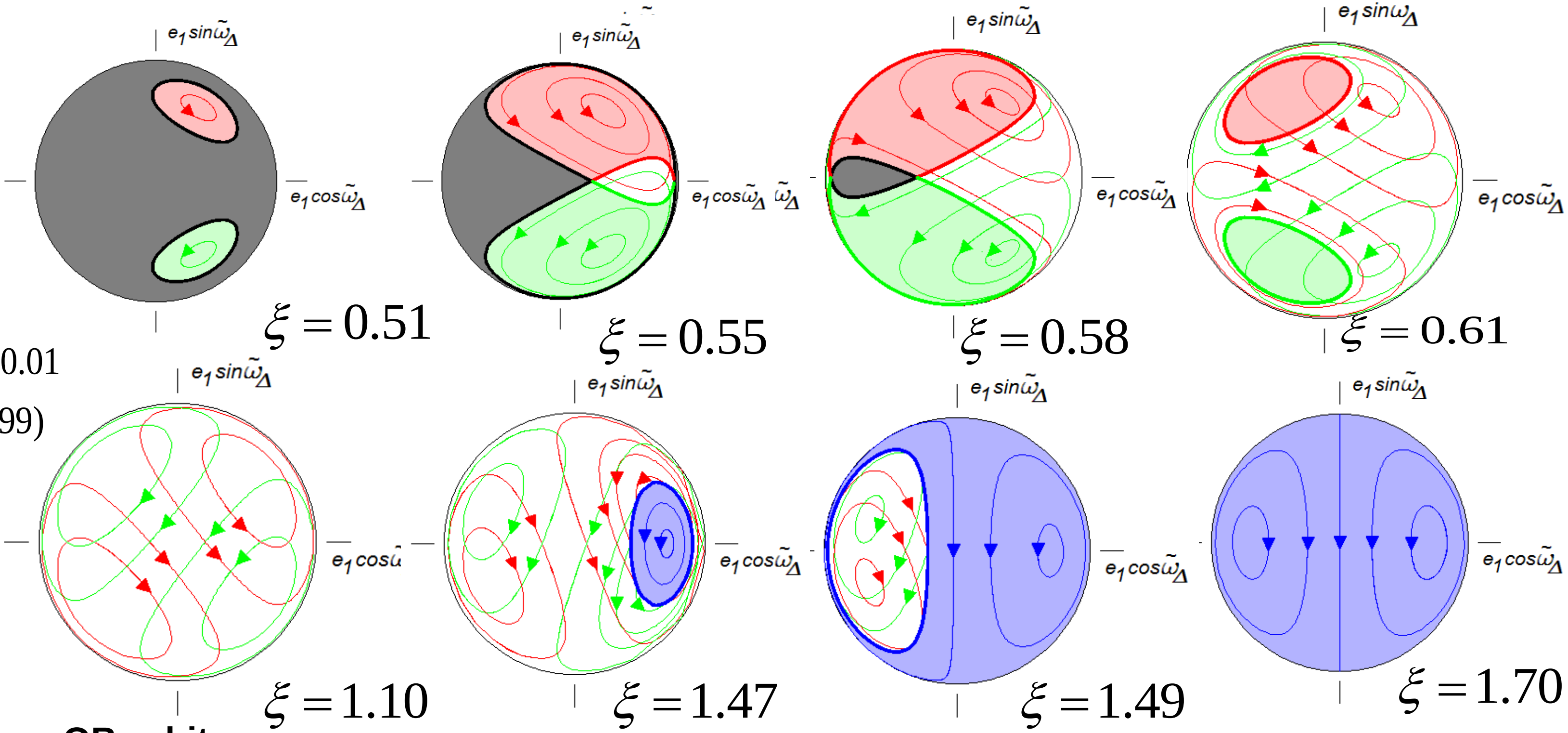
$$\bar{\mu}_1 = \bar{\mu}_2 = 0.5$$
$$\sigma = 0.2, \quad \xi = 1.20$$
$$(e_{1\max} = e_{2\max} = 0.8)$$



Black trajectories: QB-orbits
Blue trajectories: HS-orbits
Red trajectories: leading T-orbits
Green trajectories: trailing T-orbits

$$P_\Sigma = \bar{\mu}_1\sqrt{1-e_1^2} + \bar{\mu}_2\sqrt{1-e_2^2} = 1-\sigma$$

Secular evolution of co-orbital motion



$\bar{\mu}_1 = \bar{\mu}_2 = 0.5, \quad \sigma = 0.01$

$(e_{1\max} = e_{2\max} \approx 0.199)$

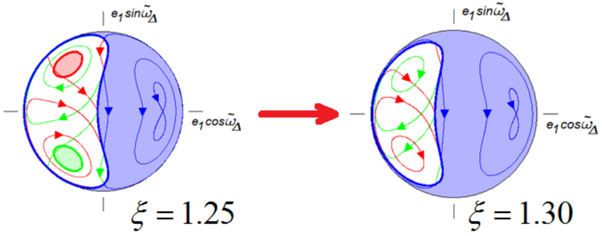
Black trajectories: QB-orbits

Blue trajectories: HS-orbits

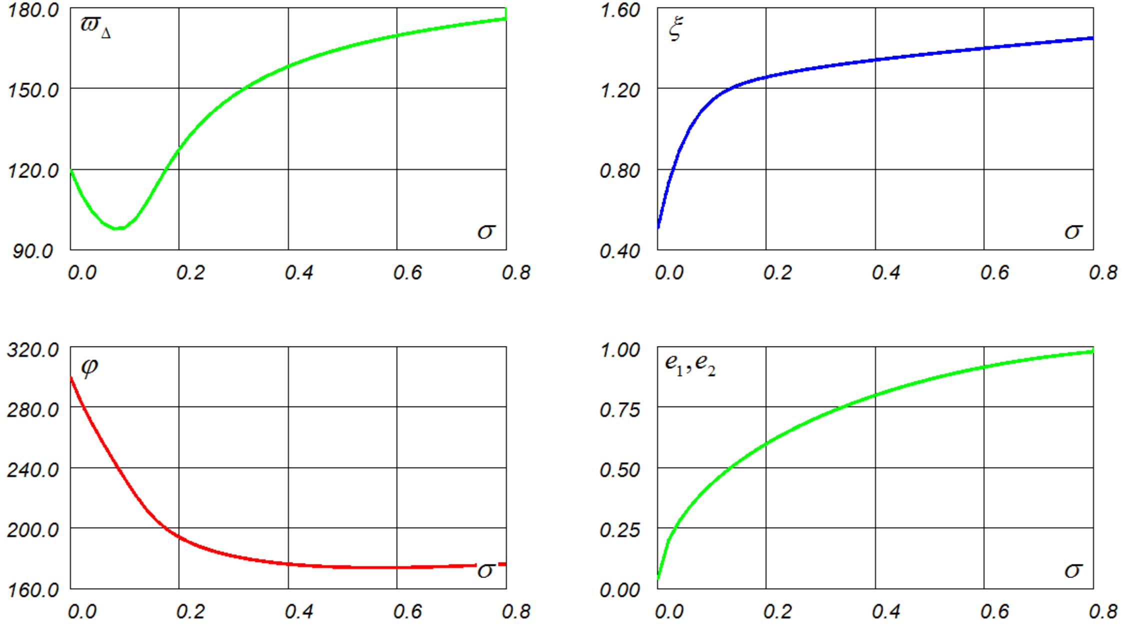
Red trajectories: leading T-orbits

Green trajectories: trailing T-orbits

Anti-Lagrangian solutions



Giuppone, C.A., Beauge, C., Michtchenko. T.A., Ferraz-Mello MNRAS (2010)



Conclusion

Adiabatic approximation is applicable for unrestricted three-body problem!

Future work

Probabilities of transitions between different regimes; “adiabatic chaos”

Open question

Can the direct co-orbital motion be transformed into the retrograde in a planar problem (and vice versa)?(Yes for non-resonant motions)

Thank you for your attention!